

Selection of Optimal Test Sequences for Identification of the Heat Dynamics of Passive Solar Components

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ABSTRACT

In this paper, methods for selection of optimal test sequences for the identification of the heat dynamics of a test cell are presented. The goal of the identification is to find the thermal characteristics of passive solar components. The efforts are part of a project called PASSYS, which aims at establishing a common basis within the European Community for determining the energy dynamics of building components.

Various measures of optimality are presented. It is demonstrated that the criteria must accomplish the purpose of the dynamic model. More specifically, it is shown that test signals that commonly are considered well suited may not be optimal if the most interesting parameters are the main dynamic characteristics, such as the overall heat transmittance and internal capacity of the building component.

INTRODUCTION

The ultimate goal of experimental design is to ensure that reliable estimates of the interesting physical parameters can be obtained by using observations obtained within a limited time interval. Experimental design for dynamic systems includes choice of input signals, sampling time, and presampling filters. Some important contributions to the theory about experimental design for dynamic systems are given in Goodwin and Payne (1977) and Zarrop (1979). A survey of experimental design for phenomenological models is found in Walter and Pronzato (1990).

Any design must take into account the constraints on the allowable experimental conditions. Some typical constraints that might be met in practice are amplitude constraints on inputs, outputs, or internal signals; power constraints on signals; limited total experimental time; limited number of samples to be analyzed; maximum sampling rate; and availability of sensors with desired accuracy and speed.

Fortunately, optimality depends only on the spectrum of the input signal, not on the actual shape, i.e., different signals having the same spectra will yield the same information about the parameters. This means that constraints most frequently can be taken into account by selecting a

given realization. This freedom also means that practical aspects, such as generation of the signal, can be taken into account. It is common practice to select binary signals, e.g., the pseudo-random binary signal (Godfrey 1980). One advantage of this signal is that it excites the system over a range of frequencies.

Theoretically, the optimal experimental design is formulated as a choice of the sequence of input variables that optimizes a suitable criterion, subject to constraints. However, it is possible only for very simple cases to obtain analytical solutions.

In this paper, a method for selecting the optimal input sequence among a set of possible input sequences is proposed. The resulting input sequence ensures optimal information about the relevant part of the thermal dynamics. However, before this information is available as estimated parameters, some more practical issues have to be considered, such as selection of sampling interval and presampling filters. These issues are not discussed in the paper (see Bloomfield [1976] and Kaiser and Reed [1977]). In the literature, some standard optimality measures are proposed. These measures and their practical meanings are discussed and, among a set of possible input sequences, the optimal sequence is found for estimation of thermal equivalent parameters belonging to a lumped model of the PASSYS test cell considered (see the next section).

However, these standard measures do not always take into account the purpose of the final model. For many applications, functions of parameters are more important. When modeling the PASSYS test cell, the UA- and CI-values, i.e., the overall heat transmittance and the internal heat capacity, are considered the most important parameters. These values are functions of the original parameters. Therefore, a method for considering application-oriented measures is proposed in the paper, and the results obtained demonstrate the importance of considering application-oriented measures.

THE PASSYS PROJECT

The application considered in this paper is related to the Commission of the European Communities (CEC) research project called PASSYS. The aim of this project is to establish a common basis within the European Com-

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munity for determining the energy dynamics of building components, especially components related to passive solar energy. Passive solar design has been recognized as an important potential for energy conservation. Many new components and systems were developed in the late 1970s and 1980s. However, very little is known about their actual thermal and solar dynamic characteristics. A further uncertainty is the unknown performance when the components are applied to buildings and exposed to variations in climate.

Within the PASSYS project, a test procedure for building components, using short-term performance data from a test cell, is developed and defined. The south wall and, for some test cells, the roof are removable. The test cell is calibrated using a highly insulated opaque south wall. Different south wall components could then be inserted in place of the calibration wall.

The test cell has a test room of 13.8 m² ground surface and 38 m³ air volume with an adjoining service room to the north accommodating measuring and air-conditioning equipment. The U-value of the envelope is less than 0.1 W/m²·K. The south wall, which is the actual passive solar system, is fixed in an insulated frame. Any kind of wall can be incorporated in this frame. A further description of the test cell is found in Wouters and Vandaele (1990).

A MODEL OF THE TEST CELL

The thermal characteristics of buildings are frequently approximated by a simple network with resistors and capacitance—see, for instance, Subbarao (1985), Hammersten et al. (1988), and Madsen and Holst (1992). In this section, such a lumped parameter model for the dynamics of the test cell is presented. The model is used for simulations and estimations in the following sections.

The dominating heat capacity of the test cell is located in the outer wall. For such buildings, the model with two time constants shown in Figure 1 is frequently found adequate. The states of the model are given by the temperature, T_i , of the indoor air and possibly the inner part of the walls with heat capacity, C_i , and by the temperature, T_m , of the heat-accumulating medium with heat capacity, C_m . H_i is the transmittance of heat transfer between the room air and the walls, while H_m is the heat transmittance between the inner part of the walls and the external surface of the

walls. The input to the system is the heat supply, Q_h , and the outdoor surface temperature, T_e . By considering the outdoor surface temperature instead of the outdoor air temperature, the effect of solar radiation is taken into account.

In state space form, the model is written

$$\begin{bmatrix} dT_i \\ dT_m \end{bmatrix} = \begin{bmatrix} -H_i/C_i & H_i/C_i \\ H_i/C_m & -(H_i + H_m)/C_m \end{bmatrix} \begin{bmatrix} T_i \\ T_m \end{bmatrix} dt + \begin{bmatrix} 0 & 1/C_i \\ H_m/C_m & 0 \end{bmatrix} \begin{bmatrix} T_e \\ Q_h \end{bmatrix} dt + \begin{bmatrix} dw_i(t) \\ dw_m(t) \end{bmatrix} \quad (1)$$

An additive noise term is introduced to describe deviations between the model and the true system. Hence, the model of the heat dynamics is given by the (matrix) stochastic differential equation

$$dT = ATdt + BUdt + dw(t) \quad (2)$$

where $w(t)$ is assumed to be a *wiener process* with incremental covariance matrix

$$\Sigma = \begin{bmatrix} \sigma_{1,i}^2 & 0 \\ 0 & \sigma_{1,m}^2 \end{bmatrix} \quad (3)$$

The measured air temperature is naturally encumbered with some measurement errors, and hence the measurement equation is written

$$T_{tr}(t) = (1 \ 0) \begin{bmatrix} T_i \\ T_m \end{bmatrix} + e(t) \quad (4)$$

where $e(t)$ is the measurement error, assumed to be normally distributed with zero mean and variance σ_2^2 .

The following parameter values have been estimated in an earlier experiment on a test cell: $H_i = 55.29$ W/K, $H_m = 13.86$ W/K, $C_i = 325.0$ Wh/K, $C_m = 387.8$ Wh/K, $\sigma_{1,i}^2 = 0.00167$ K², $\sigma_{1,m}^2 = 0.00978$ K², and $\sigma_2^2 = 0.00019$ K². Corresponding to these parameters, the time constants of the system are $\tau_1 = 3.03$ hours and $\tau_2 = 54.28$ hours. This set of parameters is used in the design of optimal test signals for the heat supply, Q_h . The outdoor surface temperature, T_e , is measured.

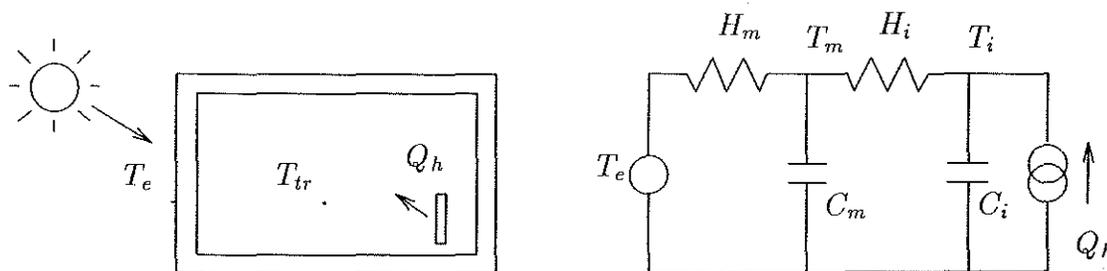


Figure 1 A model with two time constants of the test building and the equivalent electrical network.

In the analysis and discussion in this paper, the sampling time is fixed to one hour. However, the methods presented for determining an optimal input test signal could just as well be applied to obtain an optimal value of the sampling time. Furthermore, the length of the experiment has been fixed to 21 days, which corresponds to the available measurements. Zarrop (1979) suggests that experiments should have a duration of at least ten times the largest time constant. In the present case, the length of the experiment is approximately nine times the largest time constant.

Two different aspects are associated with the choice of input. One concerns the second-order properties of the signal, such as the spectrum. The other concerns the "shape" of the signal. The criteria of information described in the next section are influenced only by the second-order properties of the signal, not by the shape of the signal, i.e., it doesn't matter if the sequence is binary (oscillates between two levels) or consists of sinusoids.

DESIGN CRITERIA AND OPTIMALITY MEASURES

Assume that the qualitative part of the design has been carried out, i.e., the model structure is defined and all parameters of interest are identifiable.

In order to perform the quantitative part of the experimental design, a measure of the information achieved from an experiment is needed. It is common practice, cf. Goodwin and Payne (1977), to select a performance measure related to the expected accuracy of the parameter estimates to be obtained, in general the covariance of the parameters. The Cramér-Rao inequality gives a limit to the covariance of any unbiased estimator $g(Y)$ of θ , subject to certain regularity conditions, cf. Rao (1965),

$$\text{cov}(g) \geq (1/n)M_F^{-1} \quad (5)$$

where n is the number of observations and M_F is the Fisher information matrix (FIM), defined by

$$M_F = E_{Y|\theta} \left\{ \left(\frac{\partial \log p(Y|\theta)}{\partial \theta} \right)^T \left(\frac{\partial \log p(Y|\theta)}{\partial \theta} \right) \right\} \quad (6)$$

Y is a vector of all observations, θ is the parameter vector, and $p(Y|\theta)$ is the conditional probability density of Y for given θ . The maximum likelihood estimator $\hat{\theta}_{ml}$ for θ is asymptotically normally distributed,

$$\sqrt{n}(\hat{\theta}_{ml} - \theta) \xrightarrow{d} N(0, M_F^{-1}) \quad (7)$$

where \xrightarrow{d} denotes convergence in distribution. Equations 5 and 7 give the rationale for using the Fisher information matrix as a suitable characterization of the asymptotic parameter uncertainty.

The definition of FIM, given by Equation 6 is, under conditions of regularity, equal to

$$M_F = -E_{Y|\theta} \left\{ \frac{\partial^2 \log p(Y|\theta)}{\partial \theta^2} \right\}, \quad (8)$$

i.e., the Hessian of the log likelihood computed for the true values of the parameters. Hence, we need to derive the likelihood function for the model given in Equation 2, which is done in Åström (1980) or Madsen and Melgaard (1991).

A scalar measure of the information is then defined as

$$J = \Phi(M_F), \quad (9)$$

where Φ is a scalar function. Some standard criteria are discussed in the next section.

For the model presented previously, the set of parameters is

$$\theta' = (H_i, H_m, C_i, C_m, \sigma_{1,i}^2, \sigma_{1,m}^2, \sigma_2^2)$$

where the last three parameters describe the noise involved.

Standard Measures of Information

The most thoroughly studied criterion is *D-optimality*, defined by minimization of the determinant of M_F^{-1} , which is equivalent to minimization of

$$\Phi_0(M_F) = -\log(\det[M_F]). \quad (10)$$

A *D-optimal* design is thus obtained by maximizing the determinant of the Fisher information matrix. This criterion has a nice geometrical interpretation: The asymptotic confidence regions for the maximum likelihood estimate of θ are ellipsoids, and a *D-optimal* experiment thus minimizes the volume of these ellipsoids. An important property of *D-optimal* experiments is their independence of parameter scaling.

If only a subset of s parameters is interesting, *D_s-optimality* can be used. Assume that the first s parameters, $\theta_1, \dots, \theta_s$, are of interest ($s < p$). Then an obvious criterion to minimize is the determinant of $(A^T M_F^{-1} A)$,

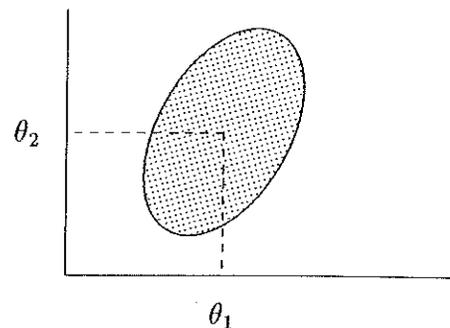


Figure 2 *D-optimal* design minimizes the volume of the confidence ellipsoid.

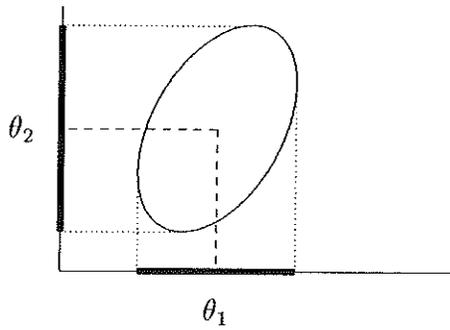


Figure 3 *G*-optimal design minimizes the relative precision of the single parameters without paying attention to the correlation structure.

where $A^T (s \times p)$ ($I_s 0$). The information matrix is partitioned in the obvious way,

$$M_F = \begin{pmatrix} M_{11} & M_{12} \\ M_{12}^T & M_{22} \end{pmatrix}, \quad (11)$$

and the matrix $(A^T M_F^{-1} A)$ can be expressed as $(M_{11} - M_{12}(M_{22})^{-1}M_{12}^T)^{-1}$. The criterion is then given by

$$\Phi_s(M_F) = -\log(\det[M_{11} - M_{12}(M_{22})^{-1}M_{12}^T]). \quad (12)$$

A criterion that only relates to the relative precision of the single parameters (without paying attention to the correlation structure) is *C-optimality*,

$$\Phi_1(M_F) = \text{tr}(H M_F^{-1} H^T), \quad (13)$$

where $H = \text{diag}(\theta_i^{-1}, i = 1, \dots, p)$. This is the same as summarizing $1/t_i^2$, where t_i is students' *t*-statistic of the *i*th parameter. This criterion is also independent of the scale of the parameters. If, instead, $H = I$ is used, the criterion is known as *A-optimality*.

E-optimal design is based on a criterion where the largest eigenvalue of M_F^{-1} is minimized. Geometrically, this corresponds to the minimization of the largest diameter of the asymptotic confidence ellipsoid for the parameters, since the semi-axes of the ellipsoid are directed as the eigenvectors of M_F^{-1} and the lengths of the semi-axes are proportional to the eigenvalues of the matrix. In other words, a design based on this criterion aims at improving the most uncertain region in the parameter space and making the confidence region as spherical as possible,

$$\Phi_\infty(M_F) = \lambda_{\max}(H M_F^{-1} H^T). \quad (14)$$

By selecting $H = \text{diag}(\theta_i^{-1}, i = 1, \dots, p)$, the criterion Φ_∞ is independent of scaling of the parameters.

Which criterion to apply depends on the demands and is often also a matter of philosophical considerations. Fortunately, an optimal design according to one of the standard optimality criteria will often show good performance according to the other criteria. This is also the case in the present simulation study.

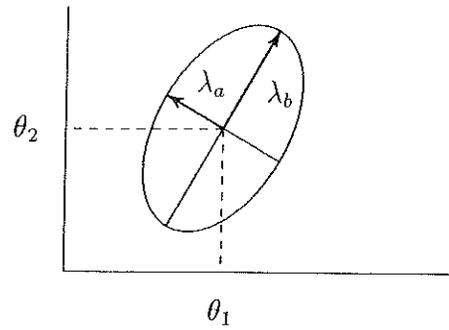


Figure 4 *E*-optimal design minimizes the largest eigenvalue of the (normalized) inverted information matrix, trying to make the confidence region as spherical as possible.

Application-Oriented Measures of Information

When dealing with physical models, nonstandard measures may be of interest, since the parameters of greatest interest might not be directly entering into the system description but might be some transformation of these parameters. When designing optimal input sequences, it is very important to take that transformation into account. This is one of the main conclusions of the present study. The UA-value and the CI-value are parameters that probably are of major interest for many applications. The UA-value is the overall thermal transmission coefficient between the inside room and the outdoor surface, and the CI-value is the internal heat capacity, defined as the amount of heat needed for raising the room air temperature by 1 K. These parameters are calculated as functions of the model parameters, and we can use the precision of these characteristic numbers as the basis for an information measure instead of using the precision of the model parameters as before. This is illustrated in Figure 5.

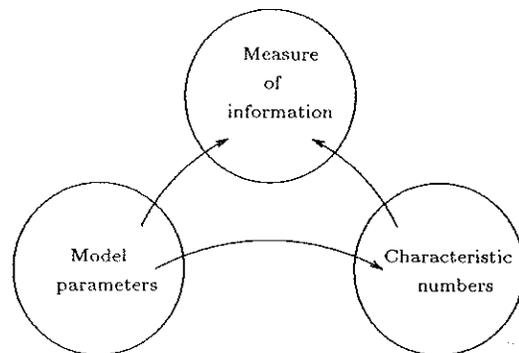


Figure 5 Diagram showing how the measure of information may be derived from the parameters of the model or from some characteristic numbers that are functions of the parameters.

Let the characteristic numbers be given as a function, $f(\theta)$, of the parameters of the model. Then the Gaussian formula can be used to approximate the information matrix in the domain of the characteristic numbers, i.e.,

$$M_F^{-1}(f) = \left(\frac{\partial f}{\partial \theta} \right)^T M_F^{-1}(\theta) \left(\frac{\partial f}{\partial \theta} \right). \quad (15)$$

Then any of the standard measures of information can be applied on $M_F(f)$. For the model specified previously, the characteristic numbers for UA and CI are calculated as

$$\begin{pmatrix} \text{UA} \\ \text{CI} \end{pmatrix} = \begin{pmatrix} H_m H_i / (H_m + H_i) \\ C_i + H_i C_m / (H_m + H_i) \end{pmatrix}. \quad (16)$$

For other applications, other transformations may be relevant. If, for instance, the model is to be used for control applications, the time constants are probably the most important parameters.

TEST SIGNALS

Usually optimal design cases described in the literature are simple enough to be treated analytically. However, this is only possible for very simple and unrealistic systems. In the present case, the analytical approach is not possible for many reasons. One such reason is that the model is rather complex (an embedded continuous-time stochastic model and discrete-time data). Furthermore, in order to have realistic input sequences, it was decided to use real measurements of the outdoor surface temperature. For these reasons, a "Monte Carlo" concept for the selection of an optimal input test signal, among a set of possible candidates, is proposed.

Different test signals for the heating power have been investigated in order to compare their properties in relation to the different optimality criteria. Most attention is paid to binary signals because of their simplicity (they are easy to implement). All binary signals switch between 0 W and 300 W and they all have equal power. The set of considered test signals is shown below. A step input and a number of PRBS sequences with different clock periods and orders have been considered. Also, a few test signals containing two sinusoids, with the same total power as the binary signals, have been tried out. The sampling time is one hour. PRBS sequences with increasing clock periods have been selected to examine the influence of increasing the period of the PRBS sequence compared to the sampling time. In other words, an optimal k in $T_{prbs} = k \cdot T_{sampl}$ is sought. The step signal is a rather low-frequency signal commonly used as the first test signal for identification; $\sin 1 - \sin 3$ consists of two sinusoids with periods $\tau_1 = 3$ hours and $\tau_2 = 54$ hours, which are close to the time constants of the system. The partition of the total power between the two sinusoids changes from 1:1 to 1:9, putting more weight on the low-frequency sinusoid. The power spectra for all the test signals are shown in the appendix.

RESULTS

A number of stochastic simulations were performed for each input sequence in order to get reasonably precise estimates for the information matrix and performance criteria. Based on experiments of this type, each input sequence is applied in 100 different stochastic simulations. Figure 6 shows the value of the D-optimality criterion for different numbers of simulations.

The results from the calculation of the optimality criteria for all the test signals are shown in Tables 1 and 2. Table 1 shows the results using standard optimality measures directly, whereas Table 2 shows the results from using the application-oriented measures.

The D_s -optimality is calculated when all parameters except the noise terms are of interest, since the noise parameters are nuisance parameters in this context. Within each column, representing an optimality criterion, the "best" test signal is the one that minimizes the value of the criterion.

It is seen from Table 2 that, according to D -optimality, the PRBS sequences have the best performance and **prbs3** proves to be the best. In other words, this test signal gives the least volume of the confidence ellipsoids of the parameter estimator. The worst performance is attached to the test signals containing the sinusoids.

If the noise terms are of minor importance, then the D_s -optimality is relevant. Also, here **prbs3** shows the best performance, but the sinusoids, especially **sin2**, have improved performance and are now better than some of the PRBS signals. The step signal is now the worst.

A comparison between the D -optimality and D_s -optimality indicates that when we are interested in the noise terms in a model, PRBS signals and not sinusoidal test signals should be used. The sinusoids are only exciting the system at frequencies around the time constants of the system, cf. the power spectrum in the appendix, which is not sufficient if the noise terms of the model are also estimated. In that case, a test signal with power in a wide range of frequencies should be used.

Looking at C -optimality, it is seen that all PRBS signals have better performance. Again, **prbs3** minimizes the criterion among all the signals. This means that if the relative precision of the parameters is most important, a PRBS signal should be used. The same tendency is shown for E -optimality, but the interpretation of this criterion is in some way different. The signal **prbs3** is the one where the maximum diameter of the confidence ellipsoids for the parameters is the least. In other words, the worst determined parameters have the highest accuracy when this test signal is used.

Furthermore, the results clearly point out that we gain from increasing the clock period of the PRBS sequence compared to the sampling period. In this case, an optimum is found for $k = 5$ in $T_{prbs} = k \cdot T_{sampl}$.

Now consider the results from using the application-oriented measures. The purpose of the measure is to focus

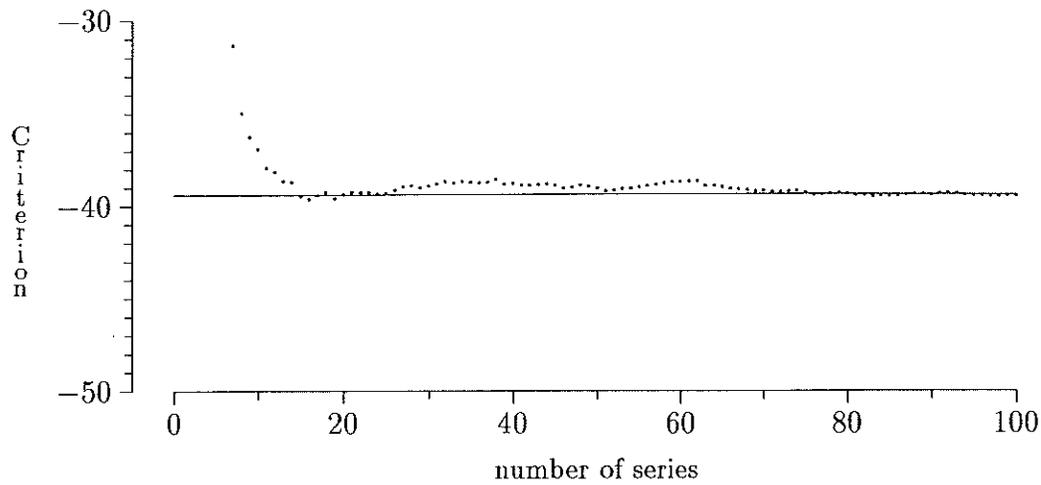


Figure 6 Plot of the D-optimality for sin1 versus the number of series included in the calculations.

TABLE 1
The Test Signals Considered

Test signal	Description
prbs1	a PRBS signal with $T_{prbs} = 1h$, order $n = 9$
prbs2	a PRBS signal with $T_{prbs} = 2h$, order $n = 8$
prbs3	a PRBS signal with $T_{prbs} = 5h$, order $n = 7$
prbs4	a PRBS signal with $T_{prbs} = 8h$, order $n = 6$
step	a step signal
sin1	two sinusoids of period $\tau_1 = 3h$, $\tau_2 = 54h$, partition of power 1:1
sin2	two sinusoids of period $\tau_1 = 3h$, $\tau_2 = 54h$, partition of power 1:2
sin3	two sinusoids of period $\tau_1 = 3h$, $\tau_2 = 54h$, partition of power 1:9

TABLE 2
Results from Calculation of the Standard Measures of Information
from Section 2 for the Different Test Sequences

Test signal	D-optimality	D_s -optimality	C-optimality	E-optimality
prbs1	-41.74 (4)	7.786 (7)	0.327 (4)	0.291 (4)
prbs2	-42.42 (3)	7.121 (4)	0.321 (3)	0.289 (3)
prbs3	-43.11 (1)	6.360 (1)	0.308 (1)	0.278 (1)
prbs4	-42.82 (2)	6.486 (2)	0.319 (2)	0.287 (2)
step	-40.94 (5)	8.760 (8)	0.451 (5)	0.426 (5)
sin1	-39.29 (7)	7.328 (5)	1.551 (8)	1.474 (8)
sin2	-39.71 (6)	7.087 (3)	1.304 (7)	1.232 (7)
sin3	-39.18 (8)	7.609 (6)	1.231 (6)	1.155 (6)

TABLE 3
Results from Calculation of the Application-Specific Measures of Information from Section 2 for the Different Test Sequences

Test signal	(UA)-optimality	(CI)-optimality	$D_{UA,CI}$ -optimality
prbs1	0.3068 (8)	485.0 (8)	70.14 (8)
prbs2	0.2596 (7)	268.1 (7)	45.55 (7)
prbs3	0.1180 (3)	164.3 (6)	15.80 (5)
prbs4	0.0907 (2)	132.2 (4)	10.71 (3)
step	0.0374 (1)	67.04 (1)	2.507 (1)
sin1	0.1636 (6)	138.1 (5)	18.89 (6)
sin2	0.1596 (5)	94.78 (3)	13.52 (4)
sin3	0.1453 (4)	71.80 (2)	10.03 (2)

on the UA- and CI-values. It is clearly seen from Table 2 that the step signal now gives optimal information. Among the sinusoids, the sequence that has the most weight on the low-frequency part has the best performance and, among the PRBS signals, the one with the largest clock period is the best. In summary, it is concluded that the signals that have a major part of the variation at low frequencies are optimal for identification of the UA- and CI-values. This corresponds nicely to the fact that these values mostly affect the frequency response function for lower frequencies.

Probably the most important lesson learned is that signals that are commonly considered generally good input signals, which is the case for PRBS signals, may not be the optimal input signals when the purposiveness of the model is taken into account by using application-oriented measures. Furthermore, the above results confirm the well-known rule of thumb: if a parameter is of special interest, design the input signal such that a major part of the signal variation is around frequencies that are mostly affected by the important parameter.

CONCLUSION

In this paper the identification of the thermal and solar dynamic characteristics of the PASSYS test cell is considered. The test cell is a small building for testing passive solar components within the European countries. The dynamics of the test cell are described by a lumped model in terms of a set of stochastic differential equations. The parameters are resistances and capacities in the corresponding thermal network. In the PASSYS project, the goal is to identify reliable values of the most important parameters with minimum experimental time. This means that experimental design concepts for dynamic system identification have to be taken into account.

In experimental design, a number of issues have to be considered—the selection of optimal input sequences being very important. For very simple systems, the optimal input sequences can be determined analytically. However, for

more realistic systems, the analytical approaches fail. This paper describes a simulation approach for evaluating various input signals. It offers the possibility to select the optimal input sequence in the set of possible input sequences considered.

Relevant measures of optimality have to be defined. Several standard measures are proposed in the literature. They build directly upon the Fisher information matrix associated with the parameter estimates. However, in many applications, some parameters or functions of the parameters are more important than the original parameters. For the PASSYS case, the UA- and CI-values are considered most important. Therefore, a method to deal with application-oriented measures has been proposed.

The set of possible input signals considered contains PRBS signals, harmonic functions, and steps. These input signals are characterized in the frequency domain.

Using the standard optimality measures, the results have shown that PRBS signals give optimal information for parameter identification. This means that if all parameters are of somewhat equal importance, PRBS signals should be used. Step input seems to give the smallest amount of information.

Using the application-oriented measures, which in the present case focus on the UA- and CI-values, it turns out that the step input is optimal. This is probably due to the fact that these values represent the lowest-frequency part of the dynamics, and the step input frequency content is concentrated in this part of the spectrum.

It is a good rule of thumb to always select an input signal that excites the system at the frequencies where a change of the most important parameters mostly affects the frequency response for the system.

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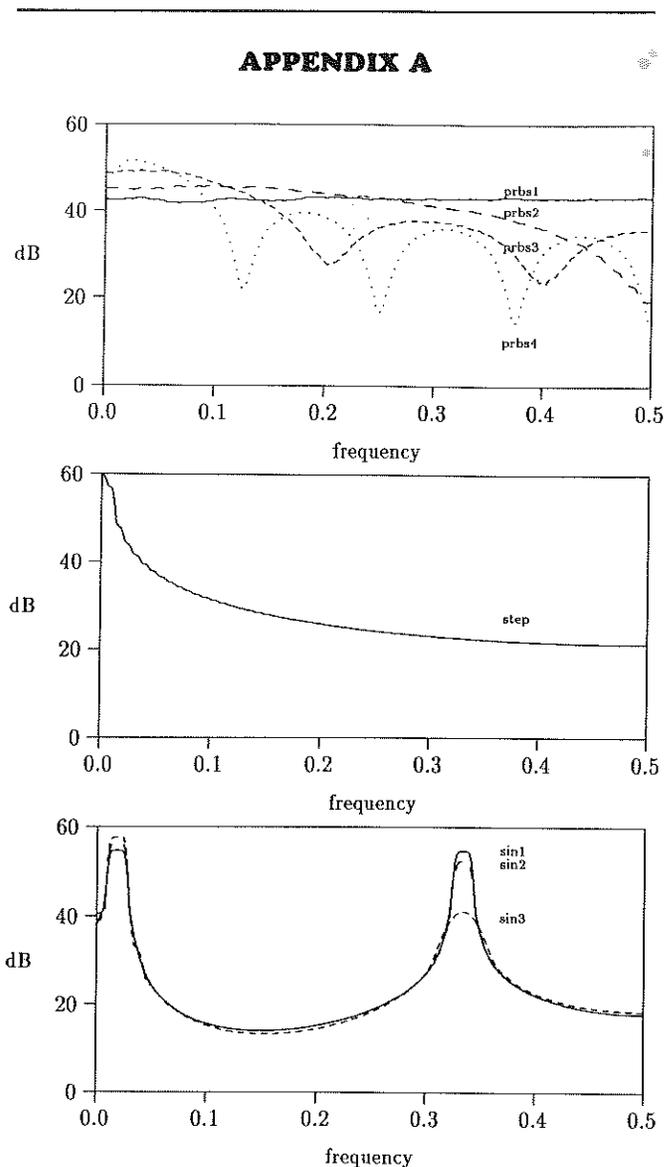


Figure A1 Estimated spectra in dB for all the test signals have been calculated by smoothing the periodogram.